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From Teachers' Predispositions to Different Socio-didactical Situations: Multiple-case Study of Expert Teachers

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Abstract

The use of textbooks and other learning resources in mathematics teaching has changed in the past decade. It is essential to investigate how this change is happening and what it means to students' learning in order to improve teaching. In this paper, we investigate more specifically how mathematics teachers have predispositions and interactions with their textbooks and other resources, and how these orchestrations and interactions interrelate with their teaching. Three expert teachers with a supervisory position in mathematics were selected, interviewed, observed and videotaped in 7, 9 and 10 double lessons, respectively. The empirical data were analysed using Siedel and Stylianides' (2018) classification of teachers' different predispositions and driving forces, and by Rezat and Sträßer's (2012) socio-didactical tetrahedron. The findings indicate that teachers' predispositions and driving forces interrelate with teaching in a very visible way in the socio-didactical tetrahedron, which is an essential aspect to articulate in teacher education programmes as well as in professional development programmes.

Keywords: Mathematics teachers' resources, Orchestration, Socio-didactical tetrahedron, Redidacticisation

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Introduction

Research on mathematics teachers' use of learning materials has evolved over the past 10–15 years (Rezat, Visnovska, Trouche, Qi, & Fan, 2018). There is increased interest in questions about how teachers use learning materials as well as whether and how new learning materials affect mathematics education and the mathematics classroom. This paper introduces a study that aims to systematically examine and analyse how three experienced and skilled teachers use mathematics textbooks and other learning resources in their teaching. The research is part of a larger research project in Denmark that investigates the use of learning resources in three subjects: L1 (Danish language), history and mathematics.

In international research (e.g., Rezat et al., 2018; Stein, Smith, Henningsen, & Silver, 2009), the use of learning materials relates to how mathematics teachers use different learning materials in orchestrating learning opportunities for students in the mathematics classroom. For instance, Remillard (2018) describes how she has gone from talking about materials to talking about resources, as the concept of resources can include a wide range of different tools, including print or digital teaching materials, videos and interactive tools (Remillard, 2018). Drijvers (2012) uses the term 'orchestration' to describe how mathematics teachers orchestrate the use of digital teaching materials in their teaching practice and how these orchestrations change over time.

The aim of this study, which is located at Læremiddel.dk, National Centre of Excellence for Learning Resources, Denmark is to develop an understanding of the use of learning resources in mathematics and to develop a subject-specific way of communicating about the use and impact of learning resources. More specifically, we are interested in:

> How do mathematics teachers interact and orchestrate with artefacts and other resources in their teaching?

By using the interview-based research of Siedel and Stylianides (2018), who identified six different approaches to teachers' selection of resources, we analyse and classify three experienced and skilled teachers from three different regions of Denmark. Based on these teachers' different approaches we analyse teachers' didactical orchestrating in general terms by using the socio-didactic tetrahedron developed by Rezat and Sträßer (2012), in order to better understand the different situations in which the teachers interact while teaching – the different socio-didactical situations.

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Teachers' Selection of Resources and the Socio-didactical Tetrahedron

The use of mathematics textbooks in classroom teaching has a special awareness in the mathematics teaching research community. How teachers use mathematics textbooks and other resources in their classroom teaching has a significant influence on mathematics education (Haggarty & Pepin, 2002; Pepin, 2018). In recent years, mathematic teachers' intention in using textbooks has been changing, moving away from a traditional use towards classroom teaching where internet resources are more integrated (Trouche, Gueudet, & Pepin, 2018).

Siedel and Stylianides (2018) introduced a classification that emerged from an interview study investigating how secondary mathematics teachers reason when they select resources. Siedel and Stylianides identified six different approaches to teachers' selection of resources: student-driven, teacher-driven, mathematics-driven, constraints-driven, resource-driven and culture-driven. The study showed that one or several of these approaches drive teachers' choices.

- A student-driven teacher considers students' needs, focuses on resource variety and which resources add value to students' mathematical learning.
- A teacher-driven teacher is driven by how they prefer to teach and their own autonomy, and they implement plans they have already developed.
- A mathematics-driven teacher select resources according to a particular topic and focuses on content appraisal.
- A constraints-driven teacher focuses on accessibility and the lack of something.
- A resource-driven teacher focuses on the characteristics of the resources themselves, their adaptability, whether they are easy to use, and whether they work.
- A culture-driven teacher is impacted by the school's culture, such as whether teachers share and collaborate.

The first three relate to classroom interactions; the last three consider other influences.

In their predispositions, teachers often make a didactic reorganisation of the learning material – a 'redidacticisation'; that is, they make a number of didactic decisions in relation to the learning material and its didactic intention, which is already present in the learning material from the publishers (Hansen & Gissel, 2017). This can of course be done in different ways.

Rezat and Sträßer (2012) have established a model for investigating artefacts and learning resources as fundamental constituents of the didactical situation. Their model, the socio-didactical tetrahedron, is based on Vygotsky's theoretical contribution to psychological tools and instrumental action (Daniels, 2016). The socio-didactical tetrahedron models the whole situation for textbooks and the use of resources from an activity-theoretical perspective (Engeström, 1998). With reference to several studies, Rezat and Sträßer argue that, when students use artefacts, the mediating role of mathematics teachers is two-sided. 'Artefacts' are considered here in a broader sense than just textbooks and include other resources such as visual representations, diagrams, language and different technologies. On the one hand, the mathematics teacher decides which artefacts to use and when to use them. On the other hand, the mathematics teacher mediates students' actual use of artefacts. At the same time, it is central to consider how a teacher's own use of the artefact influences the didactic situation and thus the teacher's instrumental action leading to the socio-didactical tetrahedron (Rezat & Sträßer, 2012).

First, Rezat and Sträßer (2012) transform the didactical triangle, also known as the 'triadic' relation in some studies (student, teacher and mathematics/subject), into a tetrahedron by introducing the artefact (Figure 1). The classic didactical triangle forms the basis of the model. Each triangular face of the tetrahedron stands for a particular perspective on the role of artefacts within mathematics education; for example, the teacher–artefact–student face considers the role of the teacher as an orchestrator and mediator, while the teacher–artefact–mathematics face illustrates the mathematics teacher's instrument-mediated activity in working with mathematics and planning mathematics lessons. Students are influenced by this activity based on the mathematics teacher's attitudes and understandings related to mathematics and mathematics didactics. The whole tetrahedron thus constitutes a comprehensive model of the didactic situation in the sense that it combines different perspectives and includes their interactions (Rezat & Sträßer, 2012).

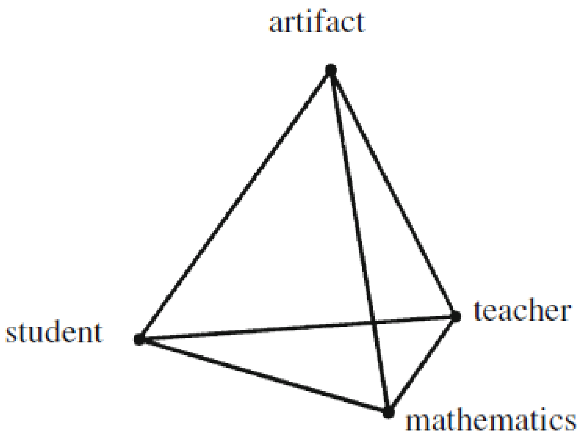

Figure 1: A tetrahedral model of the didactic situation (Rezat & Sträßer, 2012, p. 645)

Inspired by Engeström's (1998) activity-theoretical perspective, Rezat and Sträßer expand the model to the socio-didactical tetrahedron (Figure 2). In this model, the student–artefact–mathematics face represents the instrument-mediated activity of learning mathematics and is influenced by interrelated systems; for example, conventions and norms, how you 'behave' as a student, as well as the public image of mathematics.

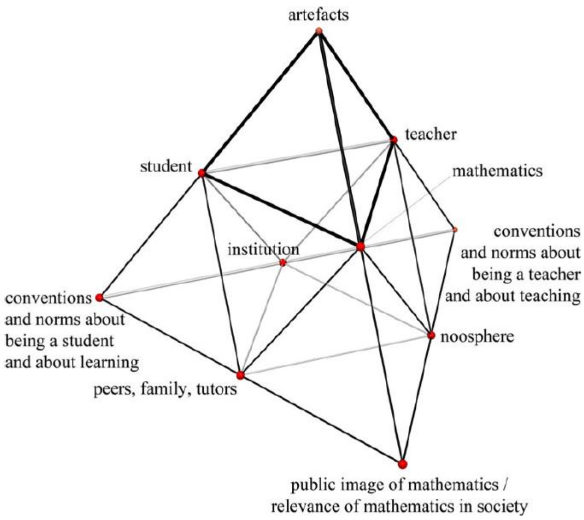

Figure 2: The socio-didactical tetrahedron (Rezat & Sträßer, 2012, p. 648)

Several studies have used the socio-didactical tetrahedron to analyse the use of mathematics textbooks (Matić & Gracin, 2016) and mathematical tasks (Freiman, Polotskaia, & Savard, 2017; Johnson, Coles, & Clarke, 2017). We use the socio-didactical tetrahedron to analyse the use of artefacts in mathematics classrooms. We now specify our research question:

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> How do mathematics teachers with different approaches to the selection of resources (their predispositions, categorised using Siedel and Stylianides [2018]) interact and orchestrate with artefacts and other resources in their teaching, as modelled in the socio-didactical tetrahedron (Rezat & Sträßer, 2012)?

Method

Three teachers were selected by the researchers (the authors of this paper) based on an intention to have expert teachers in the project. The teachers were already known to the researchers from either having been involved in earlier development and research projects or from earlier in-service training and were considered exemplarily competent before the project started. However, short distance to the schools also played a role in selection. The three teachers came from three different areas in Denmark (Southern Denmark, Funen and Zealand). The three teachers are here described as Teacher M, Teacher L and Teacher C.

Teacher M has been a mathematics teacher for 16 years. She says that mathematics has always been her primary subject, even though she also teaches science. She also serves as a mathematics supervisor at her school and has just finished her diplomas (60 ECTS) in mathematics supervision.

Teacher L has been a mathematics teacher for 17 years. She has finished more than half of her diplomas in mathematics supervision (40 ECTS point) and, like Teacher M, she also serves as a mathematics supervisor at her school. As a supervisor she spends her time as a professional expert and gives inspiration to and comments on colleagues' annual teaching plans and provides supervision of (most recently graduated) colleagues' challenges concerning students with special needs in mathematics.

Teacher C has been a mathematics teacher for 15 years. She serves as a supervisor in mathematics where, among other things, she teaches mathematics to students with autism. She finished her diplomas in mathematics supervision (60 ECTS) two years ago.

In Denmark it is normal for each school to have one specific textbook that all mathematics teachers use, which is then optionally supplemented with other resources or materials. However, the participating schools had no policy about the use of a specific textbook; the teachers could decide for themselves whether and how much to use textbooks or other resources. Teacher M reported in the interview that she used the textbook that the school provided and supplemented this with many other resources and materials. Teacher L used a standard textbook that the school provided for all the students, but it was not used in all lessons. Instead, they had a notebook in which the students themselves noted definitions and descriptions (a concept book). Teacher C had access to a standard textbook for all students but rarely used it. Instead, she made her own material by choosing tasks from different kinds of textbooks. The three teachers were observed in the same semester in, respectively, 10 double lessons (90 minutes) for Teacher M, 9 double lessons for Teacher L and 7 double lessons for Teacher C. This included 1–2 double lessons of pilot observations. For Teacher C we also observed two planning sessions that she conducted with a colleague. The observations were done with a non-participating approach (Fangen, 2008) using a handheld camera. An observation manual was created, based on ethnographic studies, which explicitly emphasises that the camera should follow the teacher and focus on what the teacher does with the textbook and/or other resources. The manual was created after the pilot study, where different angles and techniques were tried out. Immediately after each observation, a log was created containing a rough transcript where the lesson contents were described in chronological order. The observation series ended with a semi-structured interview with each teacher. The interviews were audio recorded and transcribed in full.

The research design strategy is based on a qualitative multiple-case study (Johnson & Christensen, 2014) containing three different cases: the case of M, the case of L and the case of C. The aim of these cases was to obtain a detailed and specific description of how the teachers interacted with the materials and resources. The rationale for this strategy is that case studies are often seen as a typical strategy for empirical exploration of a selected phenomenon in the context in which the phenomenon takes place, whereby the context of the phenomenon can also be included in further argument (Johnson & Christensen, 2014).

The analyses in this study have been done in two steps, both of which included theoretically defined codes. First the data was coded using Siedel and Stylianides' (2018) different classification of predispositions. The authors (the research group) first defined and then used some signs of each classification in order to categorise the three different teachers. Secondly, the authors defined codes for different signs and indicators of what was associated with the different edges and also for some of the faces and vertex corners of the socio-didactical tetrahedron (Rezat & Sträßer 2012). All the data were reviewed by at least two researchers from the group to increase the intercoder reliability.

Teachers' selections and predispositions of resources

The first analysis that is presented is made from first making a juxtaposition of the three cases (Table 1) to provide insight into which key categories clarify the teachers' way of being driven (Siedel & Stylianides, 2018) when using teaching resources. The data in the table are derived mainly from observation of the planning sessions for Teacher C and the interviews of all three teachers as well as observations of all three teachers' teaching.

	Teacher L	**Teacher M**	**Teacher C**
Use of standard textbook	Seldom uses a standard textbook	Uses a standard textbook and a digital textbook	Uses multiple textbooks
Use of other resources	The students produce their own concept book. Digital resources; e.g., GeoGebra	Digital resources; e.g., GeoGebra and digital worksheets When she uses a standard textbook or other resources it is almost always redidacticised.	Digital resources; e.g., GeoGebra Works with a colleague. They make their own worksheets.
How to plan teaching **Where does the inspiration come from?**	Starting point is her long-term planning and the curriculum. The textbook is used as a frame from which to select important concepts. Uses her own experiences to see mathematics in the world, and designs exercises/activities to make students discover it, and to get students to wonder critically themselves. How to represent mathematical concepts is her overall concern.	The students' learning process is in focus: she considers students' prior understanding while planning. The textbook is used only as an inspiration, more like a starting point. She uses a lot of materials for inspiration and emphasises the importance of the possibility to redidacticise the materials to fit the specific needs of the group of students.	Always plans with a colleague. They locate different activities in different textbooks. They redidacticise the exercises, and remove 'all the confusing matter'. Exercises/activities end up being without structure and without connection to what comes before and after in the book. Usually only one modality is represented; e.g., a table. The worksheets they make are considered useful for future planning.
A typical lesson	Begins with an introduction with a connection to prior academic knowledge and/or by using concrete materials and informal vocabulary. Afterwards, students 'talk to your partner/group' to	Starts all lessons by writing a planon the board. There are many different activities and many shifts between teaching materials, standardised or self-	Starts all lessons by writing a plan on the board. Every lesson has approximately five minutes of physical activity with respect to mathematical

	brush up on concepts. After this, a few new (or already known) concepts are presented on the board, followed by a presentation of that day's (often open-ended) activities. The lesson ends with a discussion of the concepts and the students write a note about it in their concept book.	produced. The students work in small groups and the groups are formed concerning the students' level of understanding. In situations where students wonder about the meaning of mathematical concepts, the group work is interrupted, and they discuss the meaning of the concept in a whole-class situation and momentarily M integrates the concepts into the current activity.	content. C begins the lesson by giving an oral listing of what the students are supposed to do. Then the students start working and C helps. While helping she has no focus on the rest of the students. The lesson ends without any class discussion validating students' work.
Suddenly unplanned situation, what happens?	In situations where L has ICT problems, there is one specific student who helps L. If something goes wrong, L prepares an inquiry-based mathematical activity in a flash.	In general, she considers her teaching as dynamic: things can change throughout the lesson, and she is not afraid of chaos. In a specific situation, she uses students to help other students with technical problems.	In situations where the students do not want to participate in physical activities, they have to do the activity the old-school way, with pen and paper.

Table 1: Teachers' predispositions

From Table 1, we identify how the three teachers' predispositions in their resource selection indicate what they are driven by (Siedel & Stylianides, 2018). The idea is not to generalise from these three teachers, but to understand in a more nuanced way why their resource selection differs. Throughout the observations, the mathematical content varied for the three teachers, including observations in geometry, statistics and algebra lessons.

Teacher L can be seen as mathematics-driven. The characteristic of a mathematics-driven teacher is the focus on the mathematical content. Teacher L uses her long-term planning and the curriculum as the first step to plan her lessons, and uses a textbook only as a scaffold to select the important concepts she needs for a chosen topic. She associates the selection of resources with specific mathematical topics. Her focus is on mathematical content and on consideration of how a chosen mathematical concept can be learned. The representations and explanations of mathematical concepts are very important in her planning of lesson activities. She makes a *content appraisal* (Siedel & Stylianides, 2018) concerning which representations will be the best and, based on that, she chooses her resource.

Teacher M can be seen as student-driven. Teacher M points out that the most important part of her planning and orchestration is to focus on which group of students is in the class, as well as these students' specific needs and how they learn. This agrees with the description of a student-driven teacher. Teacher M plans many different activities and there are many shifts between teaching materials – standardised, redidacticised or self-produced – during class. She emphasises the need for engaging the class and wants them to enjoy mathematics as well as to make sure that the resources fit the specific students' needs and level of understanding, especially when forming groups in class. This all points towards a student-driven teacher, which emphasises *resource variety* (Siedel & Stylianides, 2018).

Teacher C can be seen as resource-driven. Teacher C plans next week's lessons with a colleague; they end up having identical teaching plans for two different classes. Together they choose and simplify different activities from a variety of textbooks by removing 'all the confusing matter', and present the students with a much simpler version. Their choices do not consider specific students; rather, Teacher C argues that it becomes more user-friendly for her and for the students in general, and easier for the students to access. She also wants to use the simplified material again in other classes. This makes us categorise Teacher C as resource-driven. Her aim for teaching fits these characteristics: *ease of use*, *accessibility* and *fit for purpose* (Siedel & Stylianides, 2018).

How orchestration and interaction influence teaching

In the previous section we saw distinct differences in how the three teachers address how they orchestrate and interact with textbooks and other resources. Their predispositions lead to them being mathematics-driven, student-driven or resource-driven, and also influence their way of teaching, but the question is: in what way? In this section, we analyse their way of teaching by using the socio-didactical tetrahedron developed by Rezat and Sträßer (2012). Each of the three analyses will be summarised in a diagram that illustrates the parts of the tetrahedron that are the particular focus for each of the three teachers.

Teacher C

Teacher C is characterised as resource-driven. She plans lessons with a colleague by redidacticising exercises from existing textbooks. Even though mathematics is part of their planning, it is not the first priority. Additionally, they do not speak about or question particular students in their cooperative planning. Hence, the teacher interacts with the artefact even before she presents it to the students.

As an example, we present a typical relation between planning and teaching for Teacher C. Teacher C and her colleague wanted to work on inquiry-based tasks with probability. They found different tasks in different textbooks.

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They ended up choosing a specific task connecting the statistical probability with the theoretical probability for rolling the sum of two dice. The original task included the specific experiment (rolling the dice 100 times) as well as specific task where the students had to compare and reflect on the obtained frequencies for the possible outcomes with the probability of the outcomes. The textbook recommended that students use a sum of two dice chart and stated specifically in the text that this task was inquiry-based. While planning, Teacher C and her colleague reduced this specific task to an empty sum of two dice chart with no text at all (Figure 3).

Figure 3: New task designed by Teacher C and a colleague

In Teachers C's opinion, the reduced task was much easier for the students to handle. However, the students did not know that the goal was an inquiry-based task, and that the purpose was to investigate the difference between their own 100 throws of the dice and the theoretical probability. Neither the artefact nor the teacher stated this. The result was that the students aimlessly threw dice and noted the outcomes unsystematically in the chart. None of the students talked about probabilities. In our observation, Teacher C never addressed the students directly about their perspective on the mathematical issue. Teacher C did not make any remarks on the activity turning out differently than she had expected. In her mind, she had completed an inquiry-based task and had included the artefacts – a sum of two dice chart and two dice – which was her goal.

In the socio-didactical tetrahedron the main focus is the teacher's selection of artefacts; hence, the teacher–artefact edge is highlighted. Furthermore, she considers the worksheets useful for future classes. Teacher C has two other major fix points in the socio-didactical tetrahedron. She told us that she wanted to focus on inquiry-based mathematics, which has enormous which is discussed a lot in Denmark. She did not say why, but the community of other mathematics teachers – the noosphere – influenced her choices, (Rezat & Sträßer, 2012). Another major fix point in the socio-didactical tetrahedron is the institution; for example, her effort to fulfil the headmaster's decision about five minutes of physical activity for every 45 minutes of teaching. Teacher C used most of her attention in fulfilling these two fix points, which made her less aware of the students and the mathematical concepts. The analysis of Teacher C is visualised in the socio-didactical tetrahedron shown in Figure 4:

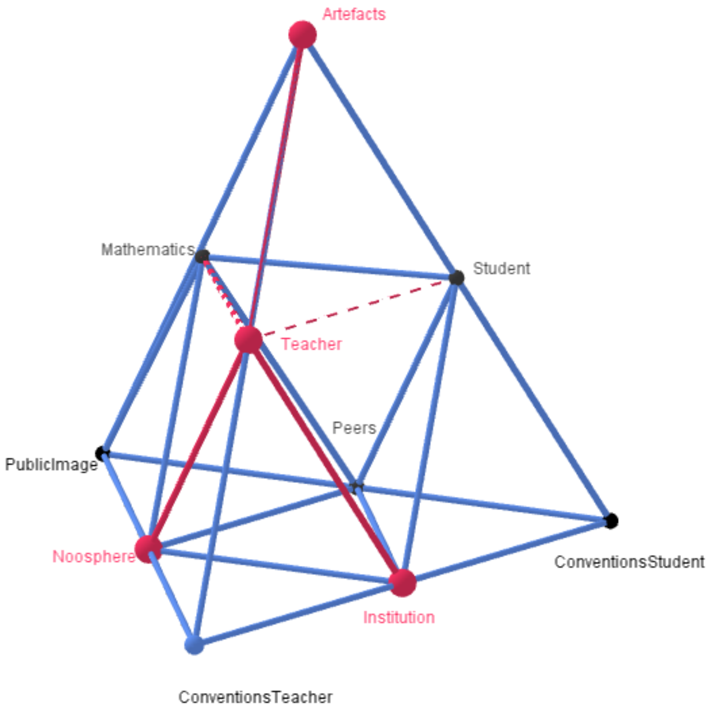

Figure 4: Socio-didactical tetrahedron for Teacher C

Teacher L

From an analysis of interviews and observations, it is clear that the subject of mathematics itself drives Teacher L. She has a mission to:

> '… Make mathematics an interesting subject. Students must feel that mathematics can be used for anything and is not dangerous. This is about being able to *see* the mathematics and the patterns and structures in everything around you. It sounds great that there is maths in everything, but there *is*!' (Interview, Teacher L)

Teacher L tried to prevent a fear of mathematics by linking the subject to real-life patterns and systems that can be explored, described and investigated through mathematics. This was apparent when students spent one mathematics lesson estimating the number of hooks in their school buildings and evaluating whether there was a hook for every student. The students then experienced that mathematics could be used in an active and authentic way to describe reality. At the same time, it was an activity where everyone could participate and contribute. The students worked with a rich variety of teaching materials during the lessons. There was work on GeoGebra, WordMat, the internet, student-produced videos filmed on smartphones or iPads, textbooks, draft booklets, concrete measurements as well as surveys outside the classroom. As a continuous learning tool, Teacher L used a concept book. The concept book was the students' own notebook where they wrote using their own words – the most important from the mathematics lessons – explanations and definitions of different concepts. In almost all of our observations of Teacher L, students were actively working with the concept book. Either they wrote, read excerpts aloud or talked to Teacher L about what was in their concept book. Teacher L largely determined the headings in the concept book, but students wrote differently in their concept book, so it was basically the students' own mathematics book. Teacher L often spent either the first or the last few minutes of the lesson asking a particular student to read a short paragraph aloud from their concept book.

In the socio-didactical tetrahedron, the main focus is the teacher's instrument-mediated activity in working with mathematics and planning mathematics lessons, hence the mathematics–teacher–artefact triangle is in focus. Teacher L, however, was also aware that not all students benefitted from working with the concept book. Some students worked best orally and had almost no text in their concept books: 'I think that [the concept book] has no value for K. Then I support him in some other way'. This reflection shows that, even though the concept book as an artefact was intended as a teaching resource to cover all students' work on mathematics, there were nevertheless differentiated considerations in relation to the students' interests and strengths/weaknesses. In both the interviews and during the lessons, she also emphasised students' conceptual learning and described concept learning as a kind of counterpole to the exercise paradigm (Skovsmose, 2001): 'Instead of just having to sit and fill in some numbers, it is the practice of communicating mathematics.' This quote is one of several examples that show she has a critical attitude towards certain teaching materials, and at the same time shows that she is also focused in the part of the socio-didactical tetrahedron (Rezat & Sträßer, 2012) where the instrument-mediated activity of learning mathematics is in focus. Hence, the balance between mathematics, artefacts and students is considered. The concept book is based both on a mathematical academic point of view, where conceptual work is more important than training assignments, and also on a notion of how to teach concepts. Thus, there is evidence that Teacher L considers the socio-cultural parts of the tetrahedron when she reflects on teaching and conventions and norms about teaching. The analysis of Teacher L is visualised in Figure 5, where the primary focus is bright red and the secondary focus is pale red:

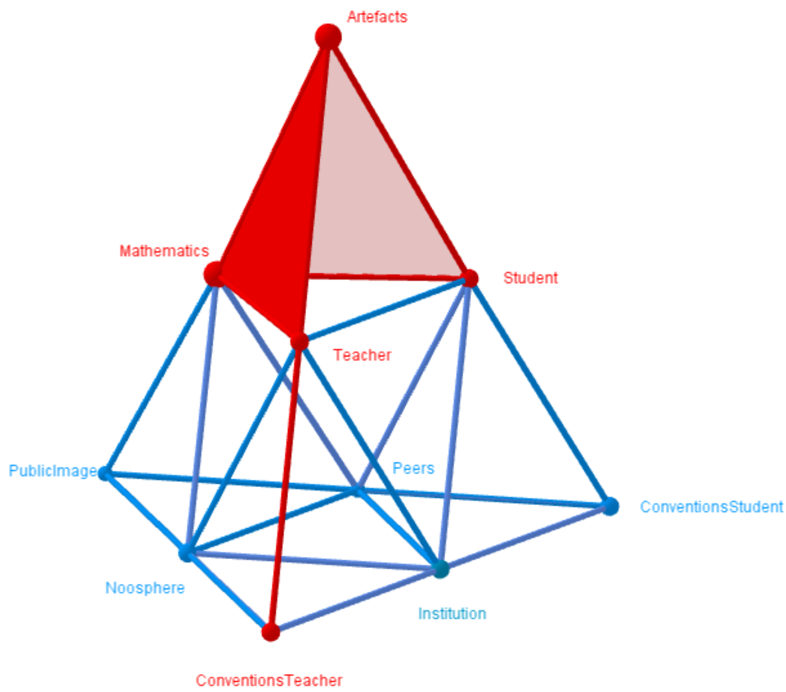

Figure 5: Socio-didactical tetrahedron for Teacher L

Teacher M

The students were always Teacher M's main concern when choosing artefacts, even if she said that her teaching was based on a textbook, which students had access to both as a paper version as well as an electronic version. The students were always considered as an important part of the didactical orchestration. Observations showed that her teaching was characterised by many transformations of artefacts and shifts between different artefacts. The shifts were typically between teacher-produced artefacts and didactically transformed parts of the textbook. She had a didactical reason for all her choices; her transformations of the textbook were all addressed to the student-perspective:

> 'Yes, when I transform exercises from a textbook it is usually because of a heavy reading-load … I don't want the students to prejudge it and say: "I am not able to do that" … we do that [shifts between different artefacts] very often during lessons; it gives the students a good flow'. (Interview, Teacher M)

She considered the students' perspective and even included herself by saying 'we'. Her main argument for transformations and shifts was students' motivation and their opportunity to engage in mathematics. She was aware of the students' need for information about these shifts and she wrote a detailed agenda on the blackboard which the students referred to several times during class. Typically, the students helped to co-design artefacts; for example, a quiz, where they were asked to make four questions based on 'facts about per cent', which was in the textbook. They had to write down one right answer and two wrong answers for each of the produced questions. Later, the students had to try their quiz outside by running to one of three cones representing the possible answers.

In another situation, she and many of the students had a technical problem while using GeoGebra: M then used the students as technical experts. She asked a student who had already solved the problem to help her and the other students. She said with a big smile on her face, 'Let us use a lifeline. We are in big trouble … I paid a lot for an expert. Pay attention, students'. She was aware of and very explicit about her role as a mediator between the textbook and the students:

> 'The textbook should not dictate, rather the students. I need to plan lessons according to the students I teach. It will never be good teaching if you insist on following the textbook … By producing it, they make it their own. It is turned upside down; it gets under their skin … they have to be in charge. You need to make them responsible'. (Interview, Teacher M)

Teacher M was also aware about the possible implications of the joint orchestration: 'I am not afraid of chaos' and 'I am not afraid that some students figure out stuff that I could not figure out myself'. In general, she saw her role as an orchestrator and a mediator, so her particular focus in the socio-didactical tetrahedron is the teacher–artefact–student triangle. However, she also addressed the conventions and norms about being a student in the socio-didactical tetrahedron when focusing on joint orchestration, students' engagement and motivation. Her way of communicating with the students shows us her thoughts about being a student in her classroom as well as her beliefs about mathematics. In the interview, she addressed this by saying: 'It is not by getting the right answer that we learn something.' She also said that students can be very creative if you let them. In her lessons, we saw multiple situations where the students talked about the meaning of concepts, and in the interview she said, 'Mathematics is primarily something you need to understand, that is why the focus on the mathematical register is important.' M's concern was usually the individual student or the small groups of students working; this also applied when she provided feedback. Her way of speaking with the students was very appreciative and she considered individual students' opportunity to engage in mathematics while using the constructed or deconstructed artefact. She considered the main purpose for the lesson and used artefacts to reinforce this. One day she made a copy of a coordinates system available for the students and commented on it in the interview: 'It helps them focus on reading the coordinates instead of using all their energy on drawing the coordinate system.' She was aware that her way of using artefacts sometimes made pauses for some students inevitable: 'It is an inevitable circumstance. If I want to speak with some students, others are waiting … maybe they speak with others about mathematics or simply listen.' She was very aware of her orchestration and the implications of the orchestrations for the students, both for her as a teacher and for her ways of introducing mathematical concepts. Additionally, it indicates a secondary focus in the socio-didactical tetrahedron that points towards interactions between students, artefacts and mathematics as well as the public image of mathematics and her understanding of the relevance of mathematics. This analysis is visualised in Figure 6, in the model of the socio-didactical tetrahedron (Rezat & Sträßer, 2012), by making her primary area bright red and her secondary area pale red:

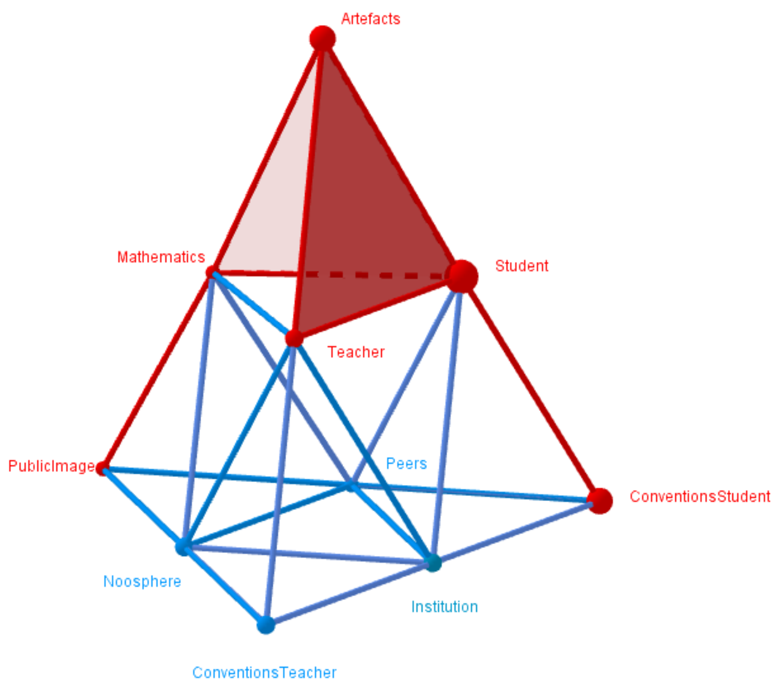

Figure 6: Socio-didactical tetrahedron for Teacher M

The Interrelation between Teachers' Different Predispositions, Focuses in the Socio-Didactical Tetrahedrons and Ways of Teaching with Artefacts

Taking a sociocultural perspective, we have analysed the role of artefacts in three cases with three expert teachers. It is obvious that the three teachers have different predispositions and are driven by different perspectives. By using the socio-didactical tetrahedron to visualise the differences it is evident that the teachers use artefacts and orchestrate teaching in very different ways. Teacher C's teaching is influenced by her selection and redidacticisation of resources and artefacts; Teacher L mediates mathematical activities for students based on her own attitudes and understanding of mathematics; and Teacher M is an orchestrator and a mediator for students' learning.

This analysis is not about excellent or less excellent teaching; rather, it is about understanding that the structures in the socio-didactical tetrahedron, the faces, edges and vertex corners, are enhanced in different ways, and that teaching with artefacts and the teacher's driving forces are closely interrelated and mutually affect each other.

By performing this analysis, obviously some aspects are more illuminated, and we are aware that there is a possibility that we thereby play down other less important aspects. We do not claim that a particular driving force leads to a specific focus in teaching as seen in the socio-didactical tetrahedron, but driving forces and predispositions interrelate with teaching in different ways. Driving forces are not static: teachers' different communities, discussions about conventions and norms, as well as changing public image, can possibly impact their driving forces. The question is therefore not about which resource or textbook a teacher uses; instead, we need to understand teachers' predispositions and driving forces. Even if the three teachers used the same textbook, their teaching would definitely be different. It could therefore be interesting to examine this in a larger study with more teachers, so that a more nuanced understanding of how the relation between teachers' predispositions and their teaching are related. In general, this case study therefore could be better substantiated with more empirical data.

Furthermore, we see a potential in the use of our case study in teacher education programmes as well as in professional development programmes, where the students will be able to use the two models in their analyses of in-service teachers to reflect on different ways of teaching in mathematical education.

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